QPI Feb 1, 2018.

1] a) $$\int |\psi|^2 dx = 1$$

$$\frac{d}{dt}\left(\quad - \quad \right) = \int \frac{\partial}{\partial t} |\psi|^2 dx$$

$$= \int \left(\psi \frac{\partial}{\partial t} \psi^* + \psi^* \frac{\partial}{\partial t} \psi \right) dx$$

↓ Sch.

(5pt) $$= \int \frac{i\hbar}{2m} \left(\psi^* \frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2 \psi^*}{\partial x^2} \psi \right) dx$$

(5pt) $$= \int \frac{i\hbar}{2m} \frac{\partial}{\partial x} \left(\psi^* \frac{\partial \psi}{\partial x} - \psi \frac{\partial \psi^*}{\partial x} \right) dx$$

$$= \left(\quad \right) \Bigg|_{x=-\infty}^{x=+\infty}$$

~~(10pt)~~ $= 0.$

b) integrate $\overset{\text{time-indep.}}{\text{Sch.}}$ (5pt)

finite ⇒ not important. (5pt)

$$\int_{-\infty}^{+\infty} \left(-\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x} + V(x)\psi(x) = E\psi(x) \right) dx$$

$$= -\frac{\hbar^2}{2m} \Delta \left(\frac{\partial \psi}{\partial x} \right) + \int_{-\infty}^{+\infty} V(x)\psi(x) = 0.$$

c) 1) finite ⇒ no discont. ⇒ smooth gradient

2) infinite ⇒ ____ ⇒ kink in ____

2] a)

~~le~~ left: $\psi = A e^{+kx}$ $\qquad \frac{d}{dx}\psi = Ak\, e^{kx}$

Right: $\quad = B \cdot e^{-kx}$ $\qquad = -Bk \cdot e^{-kx}$

(4 pt)

jump: $\Delta\left(\frac{d\psi}{dx}\right) = -(A+B)k = -\frac{2m\alpha}{\hbar^2} A$

(4 pt)

$A = B \Rightarrow k = \frac{m\alpha}{\hbar^2}$

$E = -\frac{\hbar^2 k^2}{2m} = -\frac{m\alpha^2}{2\hbar^2}$ (2 pt).

b)

ground state: (5 pt) $\qquad$ first excited: (5 pt).

c)

bosons in double well $\Rightarrow$ both in ground state

fermions $\Rightarrow$ one in gr_____,

one in 1st excited state

3] a) finite-dim: eigenvectors span the space.

infin.-dim: — do not necessarily —

b) infin-dim with ~~cont~~.

discrete spectrum: eigenvalues Real

cont. — not necessarily —

4] a)

$$L_\pm = L_x \pm i L_y \quad \text{(5pt)}$$

$$[L_z, L_\pm] = \pm \hbar L_\pm$$

$$[L_+, L_-] = +2\hbar L_z \quad \text{(5pt)}$$

b) L_z & L^2 commute

c) E_n: labelled by n

⊕ angul. mom. l, m.

→ degeneracy = $2m+1$

d).