

QPI

Feb 1, 2018.

1] a) $\int |\psi|^2 dx = 1$

$$\frac{d}{dt} (\quad) = \int \frac{\partial}{\partial t} |\psi|^2 dx$$

$$= \int \left(\psi \frac{\partial \psi^*}{\partial t} + \psi^* \frac{\partial \psi}{\partial t} \right) dx$$

(5pt) \downarrow = $\int \frac{\hbar k}{2m} \left(\psi^* \frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2 \psi^*}{\partial x^2} \psi \right) dx$ \downarrow Sche.

(5pt) \downarrow = $\int \frac{\hbar k}{2m} \frac{\partial}{\partial x} \left(\psi^* \frac{\partial \psi}{\partial x} - \psi \frac{\partial \psi^*}{\partial x} \right) dx$

$$= \left(\quad \right) \Big|_{x=-\infty}^{x=+\infty}$$

(6pt) = 0.

b) $\int_{-\infty}^{+\infty}$ integrate \checkmark time-indep. \checkmark Sche. (5pt)

\checkmark finite \Rightarrow not imposed. (5pt)

$$\int_{-\infty}^{+\infty} \left(-\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V(x)\psi(x) - E\psi(x) \right) dx$$

$$= -\frac{\hbar^2}{2m} \Delta \left(\frac{\partial \psi}{\partial x} \right) + \int_{-\infty}^{+\infty} V(x)\psi(x) dx = 0.$$

c) 1) finite \Rightarrow no discord. \Rightarrow smooth gradient
 2) infinite \Rightarrow — \Rightarrow kink in —

2] a) α

Left: $\psi = A \cdot e^{+kx}$ $\frac{d}{dx} \psi = Ak \cdot e^{kx}$

Right: $= B \cdot e^{-kx}$ $= -Bk \cdot e^{-kx}$

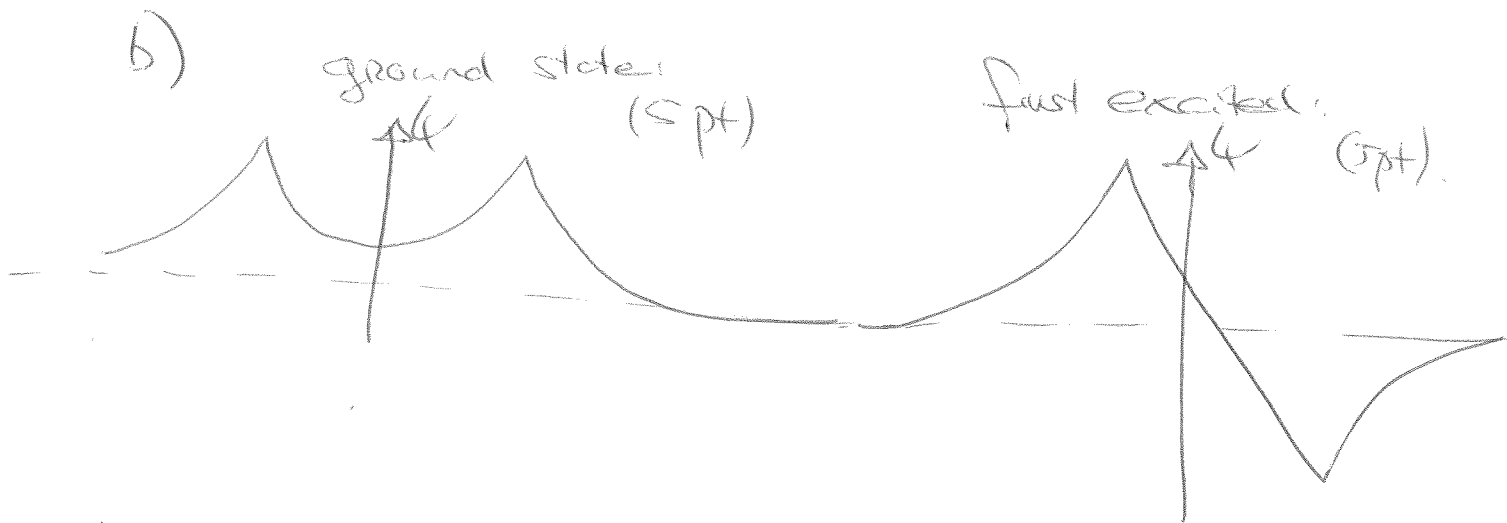
(4 pt)

jump: $\Delta\left(\frac{\partial \psi}{\partial x}\right) = -(A+B)k = -\frac{2m\alpha}{\hbar^2} \cdot A$

(4 pt)

$A=B \Rightarrow k = \frac{2m\alpha}{\hbar^2}$

$E = -\frac{\hbar^2 k^2}{2m} = -\frac{m\alpha^2}{2\hbar^2}$ (2 pt)



c) bosons in double well \Rightarrow both in ground state.

fermions \Rightarrow one in gr _____, one in 1st excited state.

